Matching of Markov Databases Under Random Column Repetitions

Serhat Bakirtas, Elza Erkip

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- Are anonymized data truly private?
- NO!
  - Correlated public data $\rightarrow$ De-anonymization!

We Found Joe Biden's Secret Venmo. Here's Why That's A Privacy Nightmare For Everyone.

The peer-to-peer payments app leaves everyone from ordinary people to the most powerful person in the world exposed.


Ryan Mac
BuzzFeed News Reporter


Katie Notopoulos
BuzzFeed News Reporter


Ryan Brooks
BuzzFeed News Reporter


Logan McDonald
BuzzFeed Staff

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- Anonymized databases containing *micro-information* shared and published routinely.
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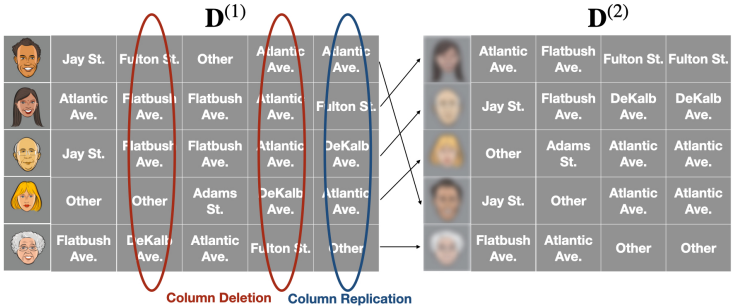
- This work: Time-indexed data, *e.g.*, financial and location data
- Synchronization errors in time-indexed data: column repetitions

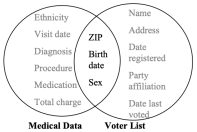


Practical Database Matching Attacks

- [Narayanan and Shmatikov, 2008]
  De-anonymization of Netflix Prize Database using IMDB data.

- [Sweeney, 2002]
  De-anonymization of medical databases using voter registration data.

- [Naini et al., 2012]
  User identification from geolocation data.



(a) Unlabeled histograms (Day 1)

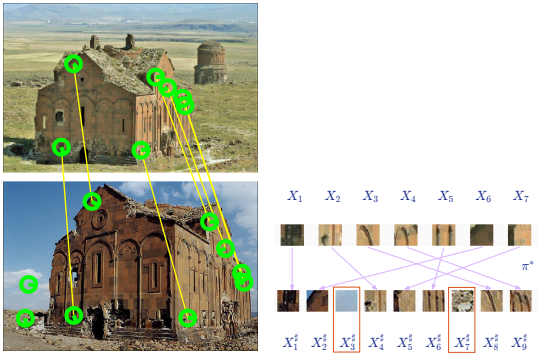
User	Location		
	Dorm.	Rest.	Lib.
?	75%	15%	10%
?	31%	30%	39%
?	15%	15%	70%
?	15%	65%	20%

(b) Labeled histograms (Day 2)

User	Location		
	Dorm.	Rest.	Lib.
John	33%	33%	34%
Jill	70%	20%	10%
Mary	15%	60%	25%
Mike	15%	20%	65%

Database Matching: Other Applications

- Computer vision [Galstyan et al., 2021]



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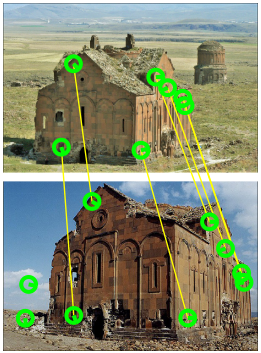

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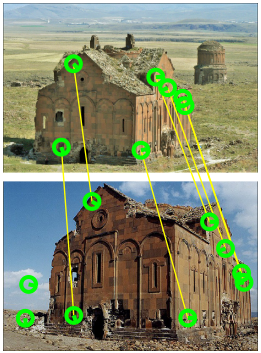

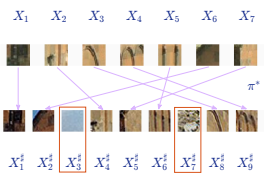

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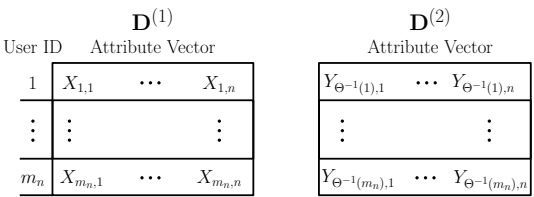
  - Single-cell data alignment [Chen et al., 2022]

Previous Works: Information-Theoretical Limits

[Shirani, Garg, and Erkip, ISIT 2019]



- Databases as $m_n \times n$ random matrices: equal no. of labeled attributes (columns)
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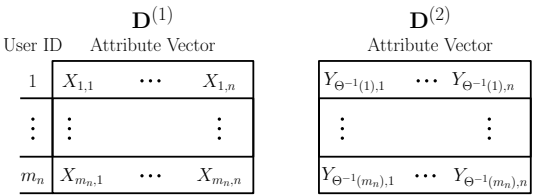

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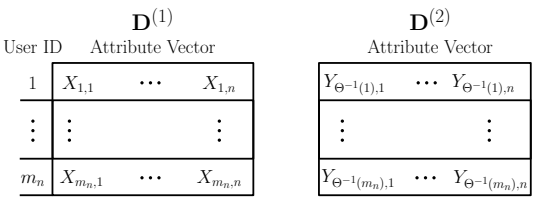

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- Successful matching: $P_e \to 0$ as $n \to \infty$
- Database matching $\Leftrightarrow$ Channel decoding

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- Objective: Given $(\mathbf{D}^{(1)}, \mathbf{D}^{(2)})$, find $\hat{\Theta}$ s.t.:

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- This allows us to
  - use information-theoretic tools,
  - work with arbitrary distributions.

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- Seeded Database Matching Under Noisy Random Column Repetitions.
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- Necessary conditions on R for database matching.

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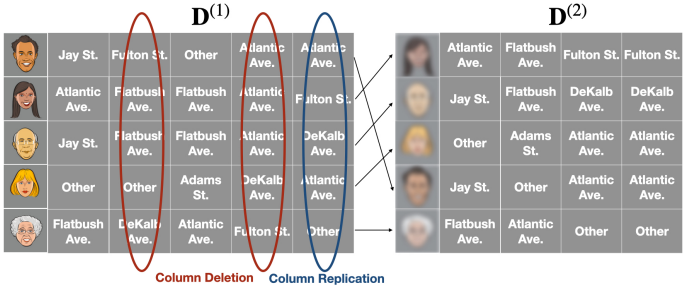
4. Repetition pattern is constant across rows.
   - Generalization to non-constant repetition pattern is possible.



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- Θ: Uniform permutation of $[m_n]$.

System Model: Continued

- Column repetition pattern: random vector $S^n = \{S_1, S_2, ..., S_n\}$ with $S_j \overset{i.i.d.}{\sim} p_S$.
  - $supp(p_S) = \{0, \ldots, s_{\max}\}$
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     - Delete the j^{th} column if $S_j = 0$.
- No noise on the entries.

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- Achievable Database Growth Rate: Given $(\mathbf{D}^{(1)}, \mathbf{D}^{(2)})$ with growth rate R, $\exists \hat{\Theta}$ s.t.:

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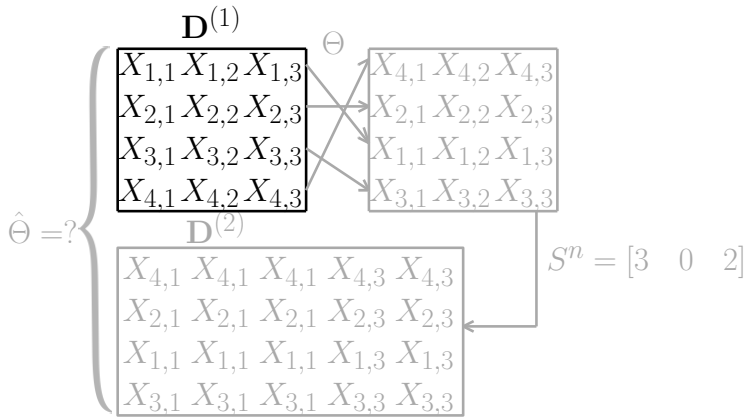
- Goal: Given $\mathbf{P}$, characterize matching capacity C.

System Model: Example

- $m_n = 4, n = 3$, S^n: repetition pattern.



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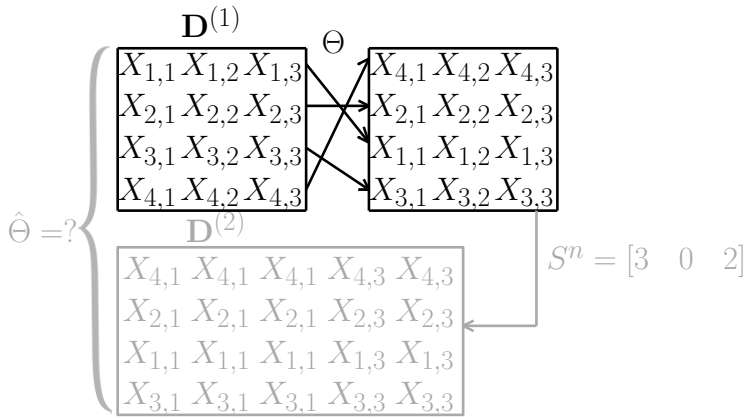

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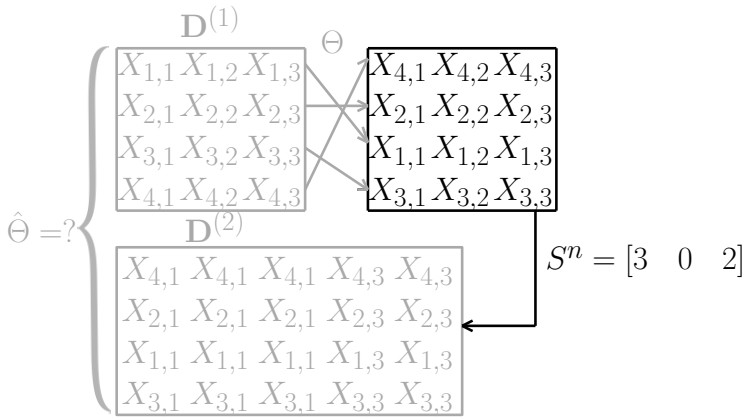

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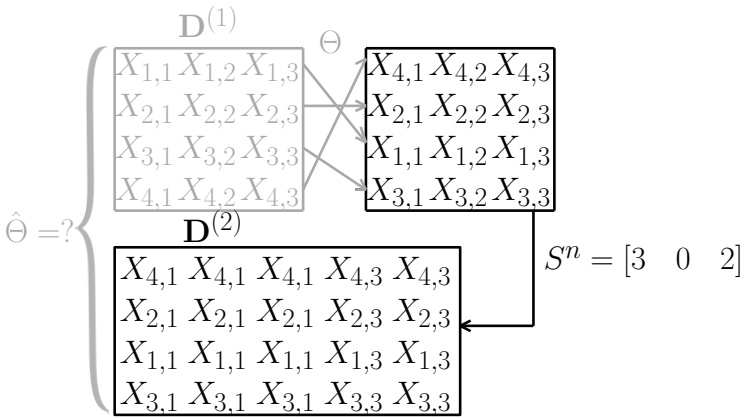

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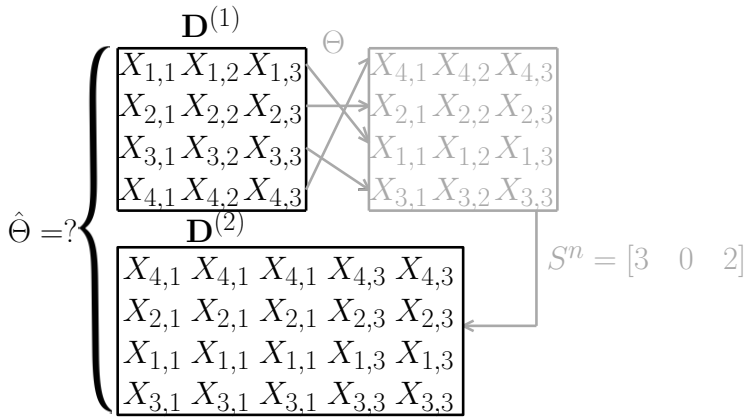

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4. If yes, how?

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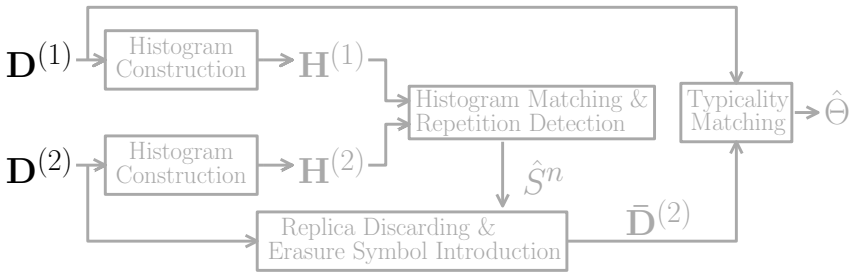

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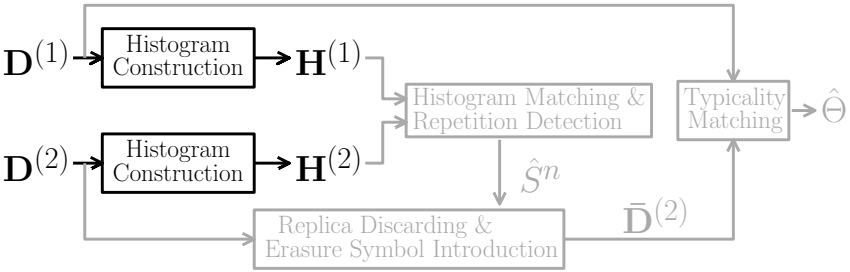

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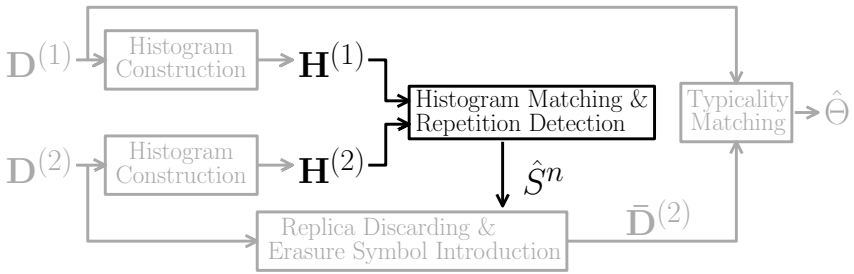

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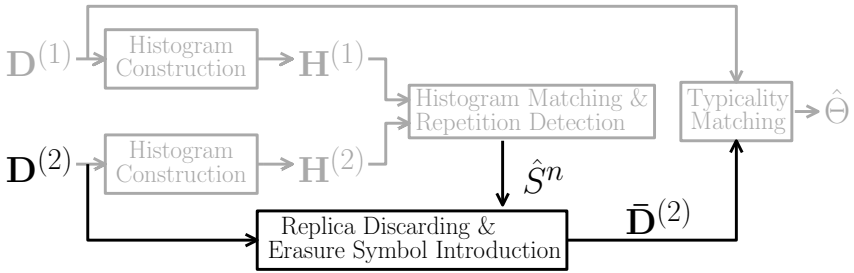

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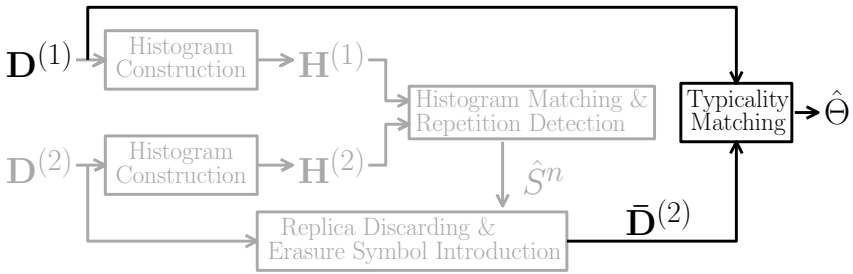

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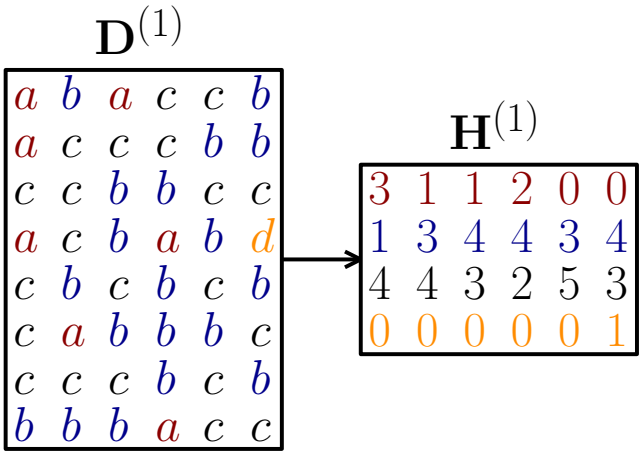

Histogram-Based Repetition Detection: Example

$\mathbf{D}^{(2)}$

a	a	a	a	a	c	b
a	a	c	c	c	c	b
c	c	b	b	b	b	c
a	a	b	b	b	a	d
c	c	c	c	c	b	b
c	c	b	b	b	b	c
c	c	c	c	c	b	b
b	b	b	b	b	a	c

$\rightarrow$

$\mathbf{H}^{(2)}$

3	3	1	1	1	2	0
1	1	4	4	4	4	4
4	4	3	3	3	2	3
0	0	0	0	0	0	1

Histogram-Based Repetition Detection: Example

$$\mathbf{H}^{(1)} = \begin{bmatrix} 3 & 1 & 1 & 2 & 0 & 0 \\ 1 & 3 & 4 & 4 & 3 & 4 \\ 4 & 4 & 3 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{H}^{(2)} = \begin{bmatrix} 3 & 3 & 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 3 & 3 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{S}^n = [2\ 0\ 3\ 1\ 0\ 1]$$

Asymptotic Uniqueness of The Histograms

Lemma

Let H_i denote the i$^{\text{th}}$ column of the histogram matrix $\mathbf{H}^{(1)}$. Then, $\Pr(\exists i, j \in [n],\ i \neq j, H_i = H_j) \to 0$ as $n \to \infty$ if $m_n = \omega\left(n^4\right)$.

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- Since there is no noise, they can be matched with the columns of $\mathbf{H}^{(2)}$.

Lemma: Sketch of Proof

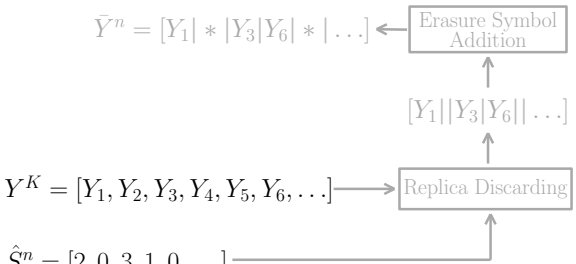
- "Collapse" the databases to binary ones.
- Union bound

$$\Pr(\exists i,j \in [n],\ i \neq j, H_i = H_j) \leq \sum_{(i,j)\in[n]^2:i<j} \Pr(\tilde{H}_i^{(1)} = \tilde{H}_j^{(1)})$$
$$\leq n^2 \max_{(i,j)\in[n]^2:i<j} \Pr(\tilde{H}_i^{(1)} = \tilde{H}_j^{(1)})$$

- $\Pr(\tilde{H}_i^{(1)} = \tilde{H}_j^{(1)}) = \sum_{r=0}^{m_n} \binom{m}{r}(1-u_1)^r u_1^{m_n - r} \Pr(\tilde{H}_j^{(1)} = r | \tilde{H}_1^{(1)} = r)$
- $\Pr(\tilde{H}_j^{(1)} = r | \tilde{H}_1^{(1)} = r)$ is $\Pr(A + B = r)$, where A and B are Binomials.
- Apply Stirling's approximation and the method of types
- Separate into two cases based on the type of Binomials
  - If not typical, summands decay exponentially
  - If typical, apply Pinsker's inequality
- Choose parameters carefully.

Replica Discarding & Erasure Symbol Addition: Example

Y^K: a row of $\mathbf{D}^{(2)}$



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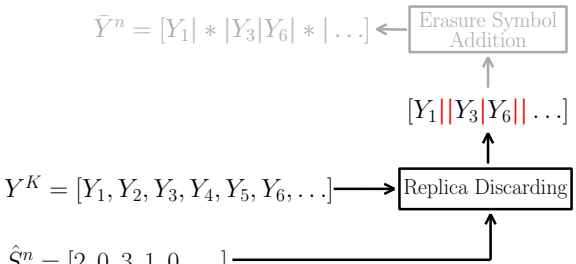

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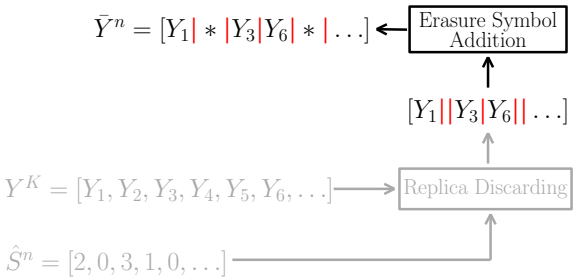

Finally, check the joint typicality of $(X^n, \bar{Y}^n)$.

Main Result: Matching Capacity

Theorem

Given a probability transition matrix $\mathbf{P}$ and a repetition probability distribution p_S, the matching capacity is

$$C = \frac{(1-\delta)(1-\gamma)}{(1-\gamma\delta)}[H(\pi) + \sum_{i\in\mathfrak{X}} u_i^2 \log u_i]$$
$$- (1-\delta)^2 \sum_{r=0}^{\infty} \delta^r \sum_{i\in\mathfrak{X}} u_i(\gamma^{r+1} + (1-\gamma^{r+1})u_i) \log(\gamma^{r+1} + (1-\gamma^{r+1})u_i)$$

where $\delta \triangleq p_S(0)$.

Main Result: Continued

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- Any deletion/replica can be converted to erasure.
- Deleted/erased columns offer temporal information.
- Replicated columns do not offer additional information.
- We have a complete characterization of the matching capacity.

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- Database Matching $\Leftrightarrow$ Channel Decoding

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- Converse result $\Rightarrow$ Insight into privacy-preserving publication of anonymized time-indexed microdata.
- Ongoing Work: Database matching when the repetition pattern is not constant across rows.

Thank you! Q&A?

Matching of Markov Databases Under Random Column Repetitions

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