Database Matching Under Adversarial Column Deletions

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  - User identities are removed: *Anonymization.*
- Are anonymized data truly private?
- NO!
  - Correlated public data $\rightarrow$ De-anonymization!

We Found Joe Biden's Secret Venmo. Here's Why That's A Privacy Nightmare For Everyone.

The peer-to-peer payments app leaves everyone from ordinary people to the most powerful person in the world exposed.

Ryan Mac BuzzFeed News Reporter
Katie Notopoulos BuzzFeed News Reporter
Ryan Brooks BuzzFeed News Reporter
Logan McDonald BuzzFeed Staff

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- Examples: Movie preferences, financial transactions data, location data, health records.

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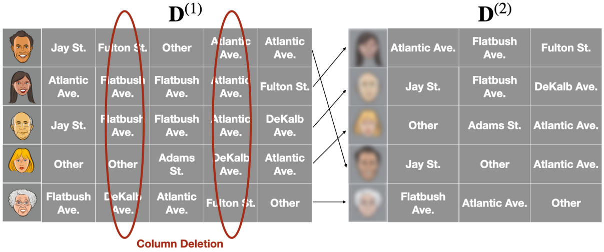
- This work: De-anonymization of time-indexed data, *e.g.*, financial and location data



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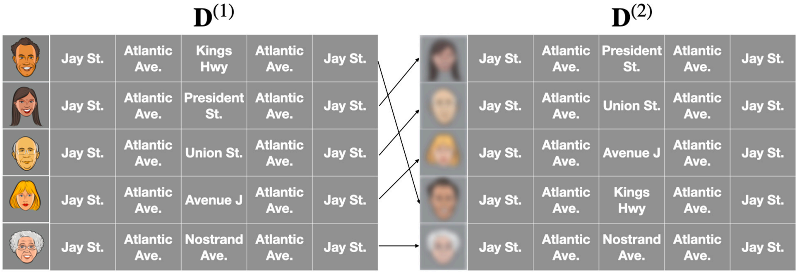
   - Intentional/Adversarial! column deletions
   - A deletion budget: Privacy - Utility trade-off

Motivation: Adversarial Column Deletions

- Some time-instances may offer more information than others.
  - *e.g.* Night-time locations reveal more private information.



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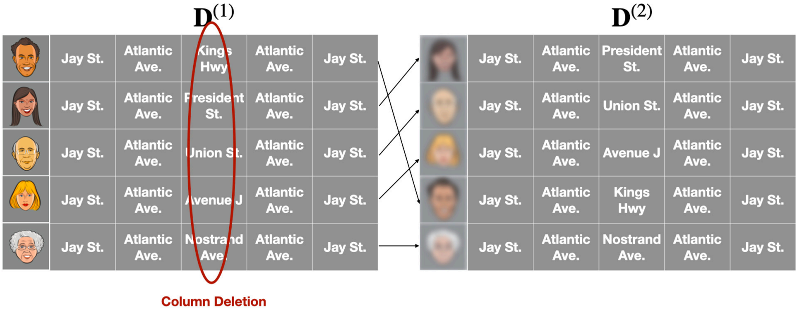

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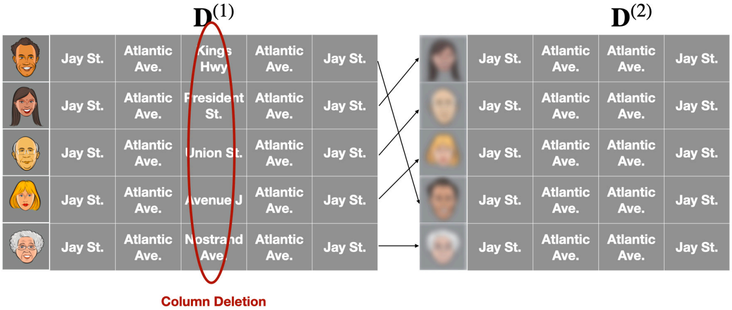
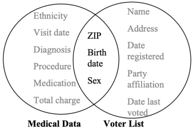


Practical Database De-Anonymization Attacks

- [Narayanan and Shmatikov, 2008]
  De-anonymization of Netflix Prize Dataset using IMDB data.

- [Sweeney, 2002]
  De-anonymization of medical databases using voter registration data.

- [Naini et al., 2012]
  User identification from geolocation data.



(a) Unlabeled histograms (Day 1)

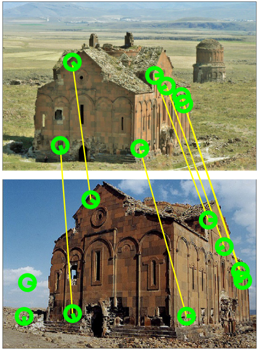
User	Location		
	Dorm.	Rest.	Lib.
?	75%	15%	10%
?	31%	30%	39%
?	15%	15%	70%
?	15%	65%	20%

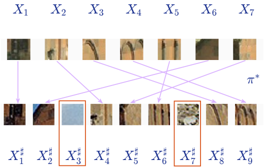
(b) Labeled histograms (Day 2)

User	Location		
	Dorm.	Rest.	Lib.
John	33%	33%	34%
Jill	70%	20%	10%
Mary	15%	60%	25%
Mike	15%	20%	65%

Database Matching: Other Applications

- Computer vision [Galstyan et al., 2021]





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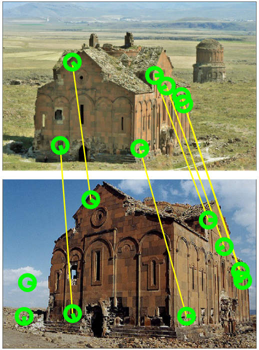

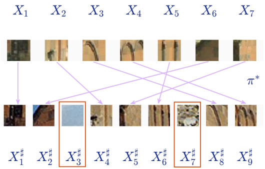

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  - DNA Sequencing [Blazewicz et al., 2002]

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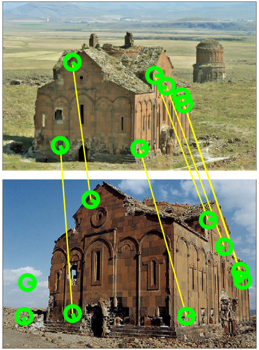

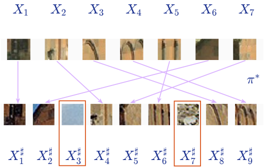

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  - Single-cell data alignment [Chen et al., 2022]

Previous Works: Information-Theoretical Limits

[Shirani, Garg, and Erkip, ISIT '19]

$\mathbf{D}^{(1)}$

User ID	Attribute Vector		
1	$X_{1,1}$	$\cdots$	$X_{1,n}$
$\vdots$	$\vdots$		$\vdots$
m_n	$X_{m_n,1}$	$\cdots$	$X_{m_n,n}$

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$Y_{\Theta^{-1}(1),1}$	$\cdots$	$Y_{\Theta^{-1}(1),n}$
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- Databases as $m_n \times n$ random matrices: equal no. of labeled attributes (columns)
    - Matching rows $\sim f_{X^n,Y^n}$: Noise-only.
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- Successful matching: $P_e \to 0$ as $n \to \infty$
- Database matching $\Leftrightarrow$ Channel decoding

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- Objective: Given $(\mathbf{D}^{(1)}, \mathbf{D}^{(2)})$, find a successful matching scheme $\hat{\Theta}$
  - Successful: $\lim_{n\to\infty} \Pr(\Theta(I) = \hat{\Theta}(I)) = 1$ where $I \sim U(1, m_n)$.

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- Achievable Database Growth Rate: Rate R is achievable if given $(\mathbf{D}^{(1)}, \mathbf{D}^{(2)})$ with growth rate R, there exists a successful matching scheme.
- Matching Capacity:

$$C \triangleq \sup\{R\colon\ R \text{ is achievable.}\}$$

Theorem (Noise-Only Matching Capacity)

In the noise-only setting, the matching capacity is given by $C = I(X; Y)$.

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1. **Random Deletions & Replications** [**Bakirtas** & Erkip, ISIT '21, Asilomar '22]
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Theorem (Seeded Matching Capacity with Repetition + Noise)

Given a seed size $\Lambda_n = \Omega(\log m_n)$ the matching capacity is $C = I(X; Y^S, S)$.

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- $\mathbf{D}^{(1)}$: $m_n \times n$ random matrix with entries $X_{i,j} \overset{i.i.d.}{\sim} p_X$.

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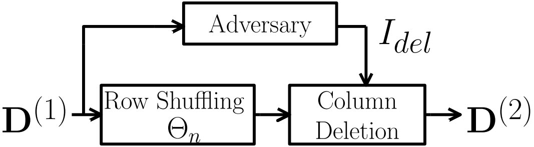
- Θ_n: Uniform permutation of $[m_n]$.
- Column deletion pattern: $I_{\text{del}} = \{i_1, i_2, \ldots, i_d\} \subseteq [n]$.
  - Chosen by an adversary after observing $\mathbf{D}^{(1)}$
  - $\delta \triangleq \frac{d}{n}$: Deletion budget
  - Identical deletion pattern across rows.

System Model: Continued



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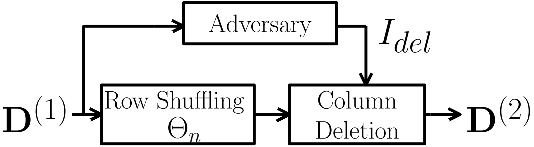

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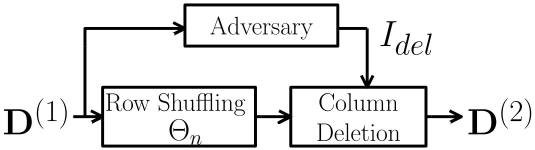

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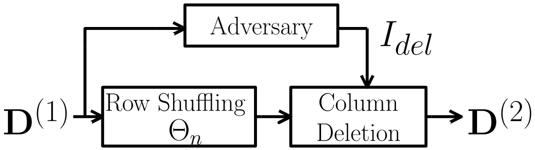

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- No noise on the entries.

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$$\Pr(\forall I_{\text{del}} = (i_1, \dots, i_{n\delta}) \subseteq [n], \hat{\Theta}_n(J) = \Theta_n(J)) \xrightarrow{n\to\infty} 1,$$

where $J \sim U(1, m_n)$.

- Adversarial Matching Capacity:

$$C^{\text{adv}}(\delta) \triangleq \sup\{R\colon\ R \text{ is achievable with deletion budget } \delta.\}$$

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- Goal: Given p_X and δ, characterize matching capacity $C^{\text{adv}}(\delta)$.

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3. Can adversarial deletion offer better privacy than the random one?

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Histogram-Based Repetition Detection: Example

$$\mathbf{D}^{(1)} = \begin{bmatrix} a & b & a & c & c & b \\ a & c & c & c & b & b \\ c & c & b & b & c & c \\ a & c & b & a & b & d \\ c & b & c & b & c & b \\ c & a & b & b & b & c \\ c & c & c & b & c & b \\ b & b & b & a & c & c \end{bmatrix} \rightarrow \mathbf{H}^{(1)} = \begin{bmatrix} 3 & 1 & 1 & 2 & 0 & 0 \\ 1 & 3 & 4 & 4 & 3 & 4 \\ 4 & 4 & 3 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Histogram-Based Repetition Detection: Example

$$\mathbf{D}^{(2)} = \begin{bmatrix} a & a & c & b \\ a & c & c & b \\ c & b & b & c \\ a & b & a & d \\ c & c & b & b \\ c & b & b & c \\ c & c & b & b \\ b & b & a & c \end{bmatrix} \longrightarrow \mathbf{H}^{(2)} = \begin{bmatrix} 3 & 1 & 2 & 0 \\ 1 & 4 & 4 & 4 \\ 4 & 3 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

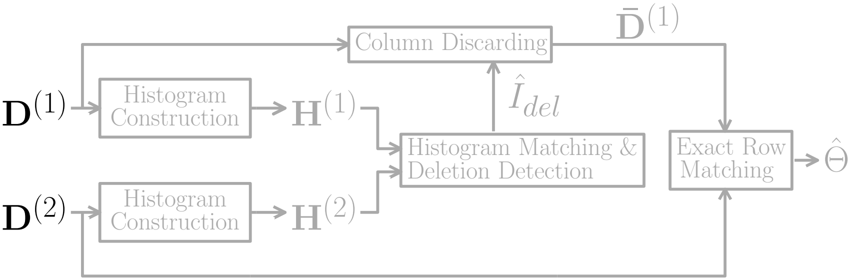
Histogram-Based Repetition Detection: Example

$$\mathbf{H}^{(1)} \qquad \mathbf{H}^{(2)}$$

$$\boxed{\begin{matrix} 3 & 1 & 1 & 2 & 0 & 0 \\ 1 & 3 & 4 & 4 & 3 & 4 \\ 4 & 4 & 3 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}} \qquad \boxed{\begin{matrix} 3 & 1 & 2 & 0 \\ 1 & 4 & 4 & 4 \\ 4 & 3 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{matrix}}$$

$$\hat{I}_{del} = [2 \;\; 5]$$

Matching Scheme



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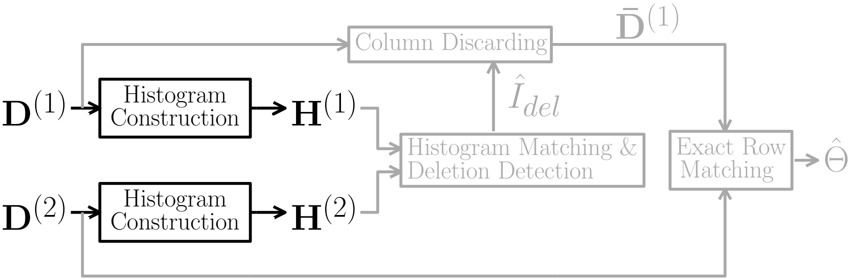

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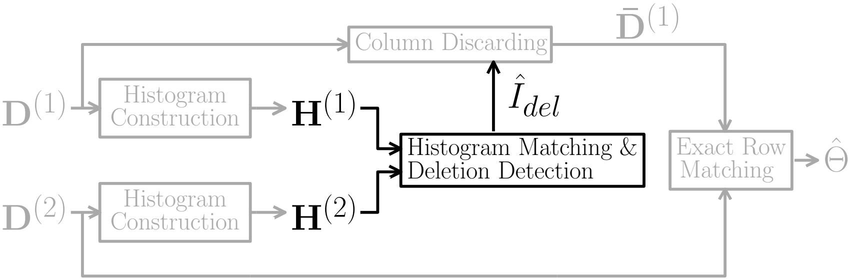

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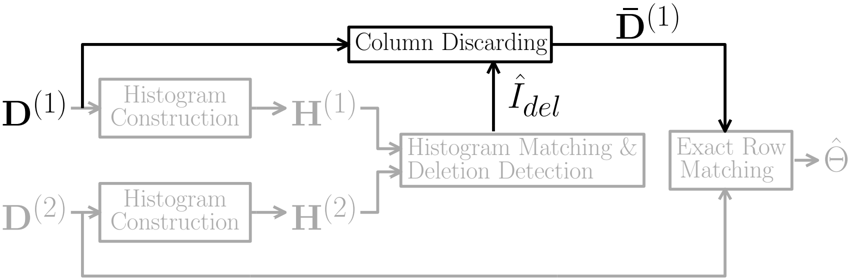

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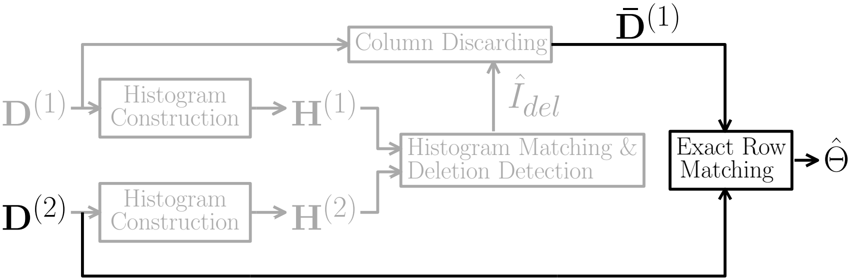

Asymptotic Uniqueness of The Histograms

Lemma

Let H_i denote the i$^{\text{th}}$ column of the histogram matrix $\mathbf{H}^{(1)}$. Then, $\Pr(\exists i, j \in [n],\ i \neq j, H_i = H_j) \to 0$ as $n \to \infty$ if $m_n = \omega\left(n^{\frac{4}{|\mathfrak{X}|-1}}\right)$.

Asymptotic Uniqueness of The Histograms

Lemma

Let H_i denote the i$^{\text{th}}$ column of the histogram matrix $\mathbf{H}^{(1)}$. Then, $\Pr(\exists i, j \in [n], i \neq j, H_i = H_j) \to 0$ as $n \to \infty$ if $m_n = \omega\left(n^{\frac{4}{|\mathfrak{X}|-1}}\right)$.

- For $R > 0$, $m_n = \omega(n^p)\ \forall p \in \mathbb{N}$.

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  - Our Result: $H_i \approx H_j$, **BUT** $H_i \neq H_j$

Main Result: Adversarial Matching Capacity

Theorem (Adversarial Matching Capacity)

Consider a database distribution p_X and an adversary with a δ-*deletion budget*. Then, the adversarial matching capacity is

$$C^{\text{adv}}(\delta) = \begin{cases} D(\delta \| 1 - \hat{q}), & \text{if } \delta \leq 1 - \hat{q} \\ 0, & \text{if } \delta > 1 - \hat{q} \end{cases}$$

where $\hat{q} \triangleq \sum_{x \in \mathfrak{X}} p_X(x)^2$ and $D(.\|.)$ denotes the KL divergence between two Bernoulli distributions with given parameters.

Main Result: Adversarial vs. Random Deletion

- Adversarial Matching Capacity

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- Random Matching Capacity [**Bakirtas** & Erkip, Asilomar '22]

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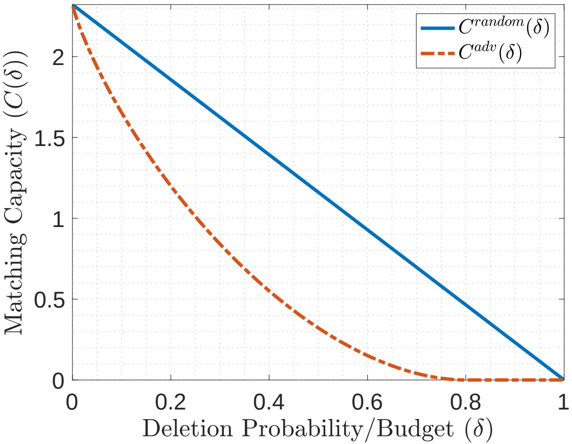
$$C^{\text{random}}(\delta) = (1-\delta)H(X)$$

  - Strictly positive!

Adversarial vs. Random Deletion: Example

$X \sim \mathrm{Unif}(\mathfrak{X})$, $\mathfrak{X} = [5]$. $1 - \hat{q} = 0.8$.



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- Histograms help us infer the deletion pattern.
- Complete characterization of the adversarial matching capacity.
- Adversarial deletions offer better privacy, compared to random deletions.
- Ongoing Work: Database matching with adversarial noise, distribution-agnostic database matching.

Thank you! Q&A?

Database Matching Under Adversarial Column Deletions

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