Distribution-Agnostic Database De-Anonymization Under Synchronization Errors

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1 Introduction
- Motivation
- Background
- This Work
- Theoretical Works

2 This Work

3 Main Results

4 Conclusion

Motivation

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  - User identities are removed: *Anonymization.*
- Are anonymized data truly private?
- NO!
  - Correlated public data $\rightarrow$ De-anonymization!

We Found Joe Biden's Secret Venmo. Here's Why That's A Privacy Nightmare For Everyone.

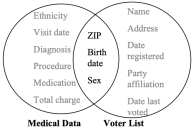
The peer-to-peer payments app leaves everyone from ordinary people to the most powerful person in the world exposed.

Ryan Mac BuzzFeed News Reporter
Katie Notopoulos BuzzFeed News Reporter
Ryan Brooks BuzzFeed News Reporter
Logan McDonald BuzzFeed Staff

Practical Database De-Anonymization Attacks

- [Narayanan and Shmatikov, 2008]
  De-anonymization of Netflix Prize Dataset using IMDB data.

- [Sweeney, 2002]
  De-anonymization of medical databases using voter registration data.



- [Naini et al., 2012]
  User identification from geolocation data.

(a) Unlabeled histograms (Day 1)

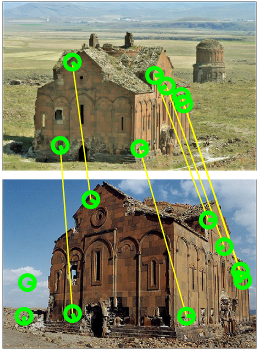
User	Location		
	Dorm.	Rest.	Lib.
?	75%	15%	10%
?	31%	30%	39%
?	15%	15%	70%
?	15%	65%	20%

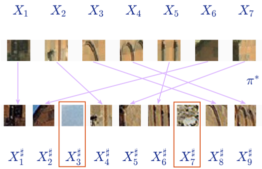
(b) Labeled histograms (Day 2)

User	Location		
	Dorm.	Rest.	Lib.
John	33%	33%	34%
Jill	70%	20%	10%
Mary	15%	60%	25%
Mike	15%	20%	65%

Database Matching: Other Applications

- Computer vision [Galstyan et al., 2021]





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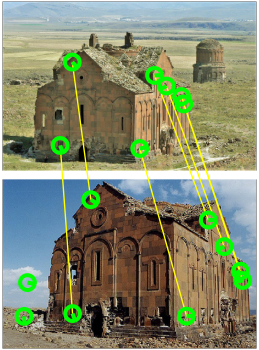

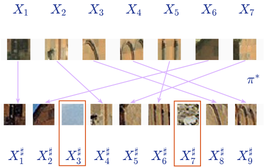

- Biological applications
  - DNA Sequencing [Blazewicz et al., 2002]
  - Single-cell data alignment [Chen et al., 2022]

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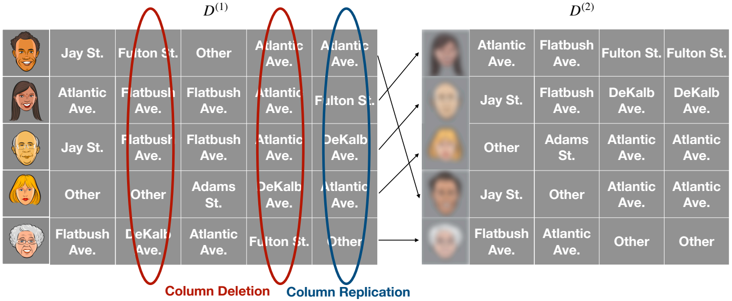
- Examples: Movie preferences, financial transactions data, location data, health records.
- This work: De-anonymization of time-indexed data with sampling errors
  - Synchronization Errors: Column deletions & replications



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1. **Random Noise** [Shirani, Garg & Erkip, ISIT '19].

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⋆ What happens when we don't know the distributions?

System Model

- $\mathbf{D}^{(1)}$: $m_n \times n$ random matrix with entries $X_{i,j} \overset{i.i.d.}{\sim} p_X$.

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- Anonymization Function: Uniform permutation σ_n of $[m_n]$.
- Column repetition pattern: random vector $S^n = \{S_1, S_2, ..., S_n\}$ with $S_j \overset{i.i.d.}{\sim} p_S$.
  - $supp(p_S) = \{0, \dots, s_{\max}\}$
  - Identical deletion pattern across rows.

System Model: Labeled Repeated Database

- Labeled Repeated Database: $\mathbf{D}^{(2)}$ is obtained from $\mathbf{D}^{(1)}$ via

$$\mathbf{D}^{(1)}$$

$$\begin{array}{c|c|c|c|c|c|}
\hline
 & X_{1,1} & X_{1,2} & X_{1,3} & X_{1,4} & X_{1,5} \\ \hline
 & X_{2,1} & X_{2,2} & X_{2,3} & X_{2,4} & X_{2,5} \\ \hline
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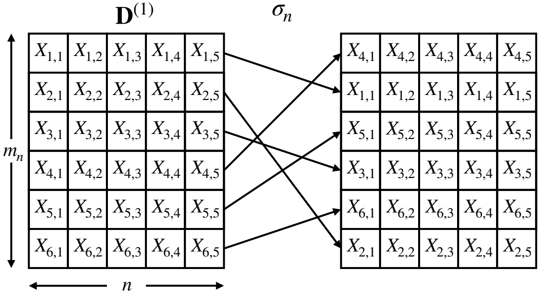
m_n & X_{4,1} & X_{4,2} & X_{4,3} & X_{4,4} & X_{4,5} \\ \hline
 & X_{5,1} & X_{5,2} & X_{5,3} & X_{5,4} & X_{5,5} \\ \hline
 & X_{6,1} & X_{6,2} & X_{6,3} & X_{6,4} & X_{6,5} \\ \hline
\end{array}$$

$$\longleftarrow n \longrightarrow$$

System Model: Labeled Repeated Database

- Labeled Repeated Database: $\mathbf{D}^{(2)}$ is obtained from $\mathbf{D}^{(1)}$ via
  - Anonymization: Row permutation via σ_n. *e.g.*, $\sigma_n = \left(\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 4 & 1 & 3 & 5 \end{smallmatrix}\right)$.



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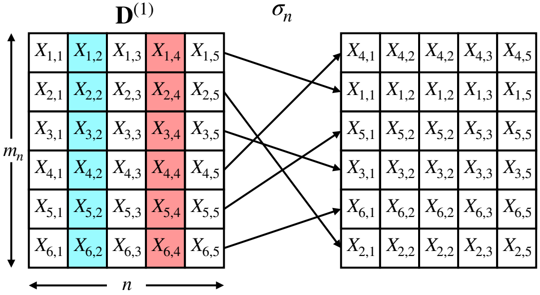
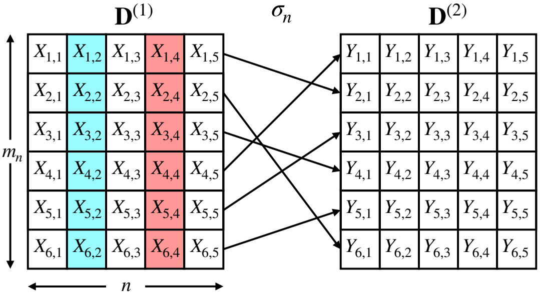
  - Synchronization Errors: Repetition via $\mathbf{S}$. *e.g.*, $S_{i,:} = [1,2,1,0,1]$, $\forall i \in [6]$.



System Model: Labeled Repeated Database

- Labeled Repeated Database: $\mathbf{D}^{(2)}$ is obtained from $\mathbf{D}^{(1)}$ via
  - Obfuscation: *i.i.d.* noise $p_{Y|X} \neq p_Y$ on the retained entries.



System Model: Continued

- Seeds: Sub-databases $(\mathbf{G}^{(1)}, \mathbf{G}^{(2)})$ consisting of Λ_n pairs of correctly-matched rows.
  - Same row generation process as $(\mathbf{D}^{(1)}, \mathbf{D}^{(2)})$.
  - Same repetition pattern S^n.
  - Λ_n: Seed size
- Successful Matching Scheme: $\phi_n : (\mathbf{D}^{(1)}, \mathbf{D}^{(2)}, \mathbf{G}^{(1)}, \mathbf{G}^{(2)}) \mapsto \hat{\sigma}_n$ with

$$\Pr\left(\hat{\sigma}_n(I) = \sigma_n(I)\right) \to 1 \text{ as } n \to \infty \text{ where } I \sim \text{Unif}([m_n]).$$

System Model: Matching Capacity

- Database growth rate R is achievable if there exists a successful matching scheme.
- Matching Capacity C is the supremum of all achievable database growth rates.
- **Our goal:** To characterize the matching capacity.

This Talk: Objectives

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   - If yes, how many seeds are sufficient?
4. Is there a capacity-wise penalty for not knowing the database distributions?

1. Introduction
2. This Work
3. Main Results
   - Replica Detection
   - Deletion Detection
   - De-Anonymization Scheme & Achievability
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Proposed Matching Scheme

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3. Typicality-Based Row Matching
   - σ_n is estimated.

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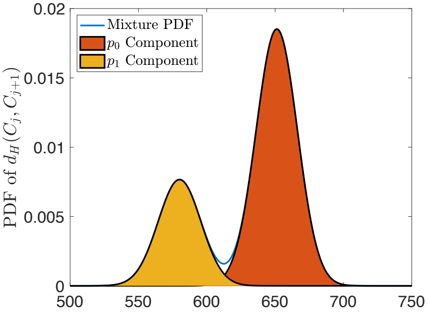
Observation 2: $p_0 > p_1$ for all $p_{X,Y}$.

Observation 3: $\alpha \xrightarrow{p} \frac{1 - p_S(0)}{\mathbb{E}[S]}$ as $n \to \infty$.

Noisy Replica Detection: An Example

$$X_{i,j} \overset{i.i.d.}{\sim} \text{Unif}([3]). \; p_{Y|X} : \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} \implies p_0 = 0.65, \; p_1 = 0.58$$

$n = 10, \; m_n = 1000, \; p_S = [0.3, 0.3, 0.4]$:



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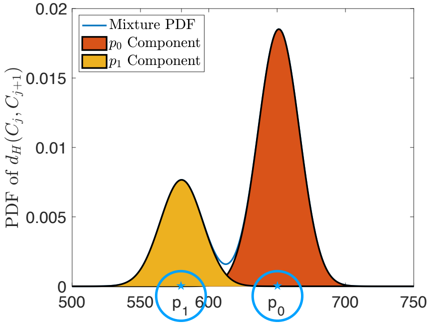

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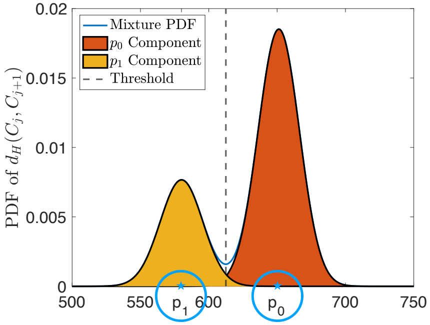

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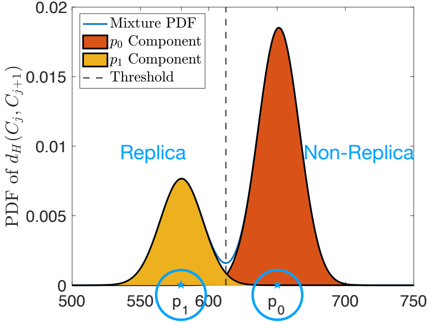

Noisy Replica Detection: Algorithm and Result

Algorithm:

1. Estimate p_0 and p_1 via a Moment Estimator.
2. Pick a threshold $\tau \in (\hat{p}_1, \hat{p}_0)$.
3. Declare C_j and C_{j+1} to be
   - noisy replicas, if $d_H(C_j, C_{j+1}) < m_n\tau$.
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Lemma (Noisy Replica Detection)

The replica detection algorithm described above has a vanishing probability of replica detection error, as long as $m_n = \omega(\log n)$.

Seeded Deletion Detection

$G_j^{(r)}$: j^{th} column of $\mathbf{G}^{(r)}$.

Observation 1:

$$d_H(G_i^{(1)}, G_j^{(2)}) \sim \begin{cases} \text{Binom}(m_n, q_1), & \text{if } G_i^{(1)} \text{ and } G_j^{(2)} \text{ correlated.} \\ \text{Binom}(m_n, q_0), & \text{otherwise} \end{cases}$$

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Observation 3: $\beta = 1 - 1/n \to 1$ as $n \to \infty$.

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Solution: Outlier detection.

Seeded Deletion Detection: An Example

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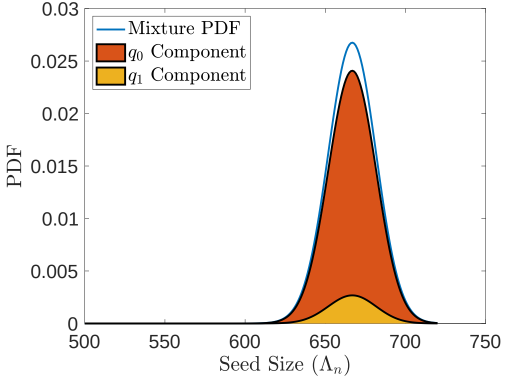
$n = 10.$



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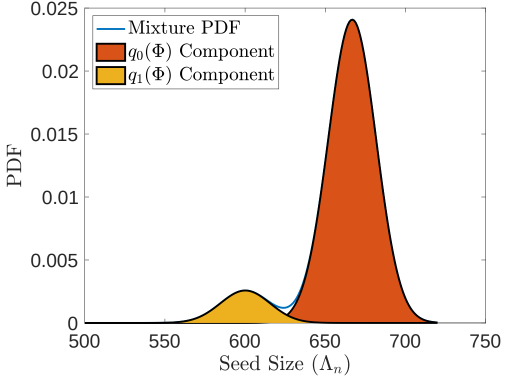
$$X_{i,j} \overset{i.i.d.}{\sim} \mathrm{Unif}([3]),\ \Phi = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.\ p^{(\Phi)}_{Y|X} : \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.1 & 0.5 & 0.4 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \implies q_0(\Phi) > q_1(\Phi)$$

$n = 10$:



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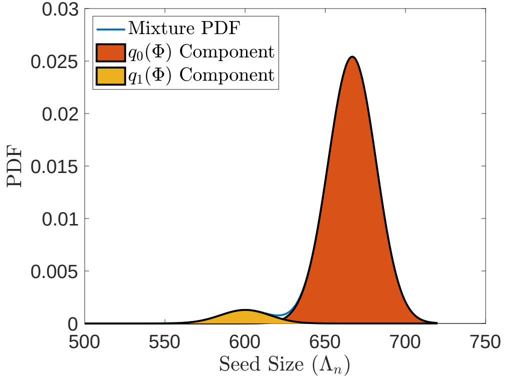

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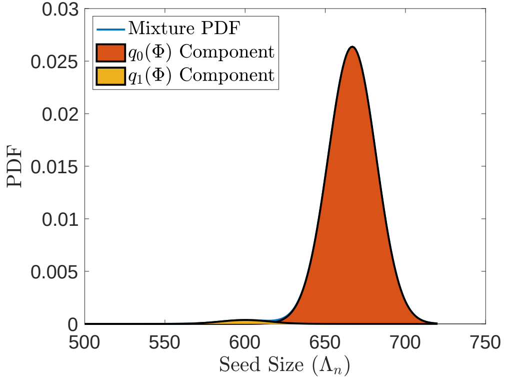

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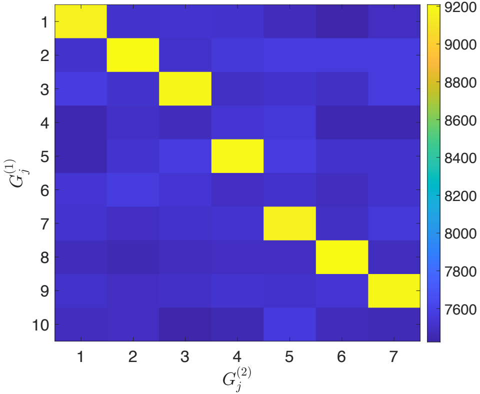
$$q_0 = 0.76,\ q_1 = 0.92,\ n = 10,\ \Lambda_n = 10^4,\ I_{\text{del}} = [4, 6, 10]$$



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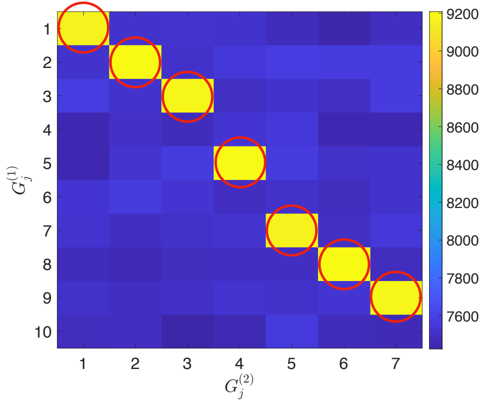

Seeded Deletion Detection: Algorithm

1. Pick a bijective remapping Φ.
2. Compute $L_{i,j}(\Phi) = d_H(G_i^{(1)}, G_j^{(2)}(\Phi))$ for all (i,j).
3. Compute the absolute deviations $M_{i,j}(\Phi)$ of $L_{i,j}(\Phi)$.
4. Declare $G_i^{(1)}$ to be
   - deleted if $M_{i,j}(\Phi) < \hat{\tau}_n \; \forall j \in [K_n]$
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Lemma (Seeded Deletion Detection)

For $\Lambda_n = \omega(\log \log m_n)$, the described deletion detection algorithm is successful.

De-Anonymization Scheme & Matching Capacity

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Given $\Lambda_n = \omega(\log \log m_n)$, the distribution-agnostic matching capacity is

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- Same as the *distribution-aware* matching capacity!

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Ongoing Work: What happens in the non-asymptotic regime?

Thank you! Q&A?

Distribution-Agnostic Database De-Anonymization Under Synchronization Errors

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NYU WIRELESS

System Model: Performance Criteria

- This work: Near-perfect matching

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$$\Pr\left(\hat{\sigma}_n = \sigma_n\right) \to 1 \text{ as } n \to \infty.$$

  - Correlated Gaussian entries.
  - No synchronization errors.

Seeded Deletion Detection: Remapping of $\mathbf{G}^{(2)}$

$$\Phi = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$\mathbf{G}^{(2)} = \begin{bmatrix} a & a & a & a & a & c & b \\ a & a & c & c & c & c & b \\ c & c & b & b & b & b & c \\ a & a & b & b & b & a & c \\ c & c & c & c & c & b & b \\ c & c & b & b & b & b & c \\ c & c & c & c & c & b & b \\ b & b & b & b & b & a & c \end{bmatrix} \qquad \tilde{\mathbf{G}}^{(2)}_{\Phi} = \begin{bmatrix} c & c & c & c & c & b & a \\ c & c & b & b & b & b & a \\ b & b & a & a & a & a & b \\ c & c & a & a & a & c & b \\ b & b & b & b & b & a & a \\ b & b & a & a & a & a & b \\ b & b & b & b & b & a & a \\ a & a & a & a & a & c & b \end{bmatrix}$$